

# Preparing for University Calculus

Prepared by the APICS Committee  
on Mathematics and Statistics

Edited by Robert Dawson

May 9, 2007

Disclaimer: This booklet is intended to give prospective students an idea of what a typical introductory calculus course at a university in the Atlantic region is like. Readers should note, however, that there are differences between calculus courses at different universities, and even within universities.

There is much important material in high school mathematics courses that is not reviewed in this booklet - to name only a few topics, probability, linear algebra, and coordinate geometry of conic sections. Omission of a topic from this booklet is not intended as an indication that it is of lesser importance than other topics.

This booklet is not intended as a textbook or achievement test. Moreover, we do not advocate any particular method of teaching the material contained herein.

Please send all error reports and suggestions for improvement to

©2002 by the Atlantic Provinces Council on the Sciences Mathematics and Statistics Committee

This booklet may be freely reprinted for educational and private use. For details see page 9.

# 1 Abo

## 1.1 What math will I need in university?

The answer to this depends on what you are taking, and you should check with the university or universities that you are thinking of attending. As a general rule, however, if you are planning to get a degree in science, you will probably need one or more courses in calculus (some universities permit programming or statistics instead in some cases). If you are taking courses in the social sciences or life sciences, you will probably need one or more courses in statistics; and degrees in physics, engineering, astronomy, mathematics, or computing science (and some

by  $\frac{1}{2}$ . Given such an expression, calculus allows us to find expressions for the integral and derivative of the function, when they exist.

### 1.3 Why is calculus important?

In the sciences, many processes involving change, or related variables, are studied. If these variables are linked in a way that involves chance, and significant random variation, statistics is one of the main tools used to study the connections. But, in cases where a deterministic model is at least a good approximation, calculus is a powerful tool to study the ways in which the variables interact. Situations involving rates of change over time, or rates of change from place to place, are particularly important examples.

Physics, astronomy, mathematics, and engineering make particularly heavy use of calculus; it is difficult to see how any of those disciplines could exist in anything like its modern form without calculus. However, biology, chemistry, economics, computing science, and other sciences use calculus too. Many faculties of science therefore require a calculus course from all their students; in other cases you may be able to choose between, say, calculus, statistics, and computer programming.

It should be understood that there is more to mathematics than calculus. Linear algebra, probability, geometry, and combinatorics are just some of the other branches of mathematics, introduced in schools, that are important at the university level. And problem-solving skills, which cut across all branches of mathematics, allow the mathematics to be applied to other subjects.

## 1.5 What is taking university calculus like?

You will probably find that university calculus is faster-paced than your high-school courses. At most universities the lecture sections will be bigger – possibly over a hundred students – and the professor will not be able to slow the class to the pace of the slowest students. You – and you alone – will be responsible for handing work in on time, and for being present for classes, tests, and exams.

This is not a course that can be passed by just memorizing everything; you have to understand it. This won't happen instantly, and the instructor cannot make it happen; you have to do that yourself, and be an active participant in the course. If you make this effort consistently through the term, you will find that it pays off.

The course material consists of a rather small number of big ideas, and a moderate number of formulae you will need to know, not hundreds of short cuts and special rules. A common mistake, especially with word problem

center person, etc. what it is that you don't understand. ("Everything is not helpful. On the other hand, do not go along demanding that the instructor tell you how to answer question 11 and refusing to listen to any explanation of why it's done that way.

- Finally, **your knowledge will be tested** on quizzes, midterms, or final examinations. The exams will probably make up the majority of your grade. If you have handed in copied, half-understood assignments, you will lose more marks at exam time than you gained in the short term. But if you kept up with the lectures and assignments, making sure you understood each bit before going on, you will probably do well.

If you find yourself falling behind, you will have to catch up. Don't panic - catching up is not impossible. There are various resources to help.

- **Yourself.** If you are falling behind and not putting plenty of time in studying - say five or six hours outside class per week *for each course* - the solution may be as near as your desk and textbook.
- **Your textbook** contains hundreds of worked examples, and thousands of problems. Usually, about half of the problems have answers in the back of the book; and you may be able to buy a study guide that shows the working of those problems in more detail. You can also get other books such as "Schaum's Outline of Calculus" containing more worked problems.
- Your instructor will have **office hours** during which you can go for help. Try

- Your university or student union may well have **workshops** on effective studying, effective note-taking, exam nerves, etc. There will also be counselling available for other problems that might interfere with your study.

## 1.6 HELP! I have to write a placement test!

Many Atlantic-area universities have found that incoming students are not uniformly well-prepared for calculus. Many students are very well prepared, and ready to start the usual calculus curriculum; but some are not. As a result, placement tests are common, and at some universities, if you do not pass the placement test, you will be required to take a remedial course covering important material from the high school curriculum **before you can take calculus**. At other universities you may just be advised to take the remedial course, or allowed into a slow-moving section that takes two semesters to cover the first semester's material.

This is to stop students from starting calculus without adequate preparation and then falling behind and failing. Experience has shown that most students who start calculus without adequate preparation do fail.

The test is often multiple-choice so that it can be graded as quickly as possible. It will generally be closed-book and written without a calculator (see the next section). Because success in first-year calculus requires a fairly high level of preparation, and because the test covers some very elementary (but important) material, the pass mark may be higher than 50%. You will probably have to write the test at or before the beginning of term; find out ahead of time what the rules are at the university you plan to attend.

Some sample questions for one such placement test may be found at <http://conway.math.unb.ca/Placement/>. Some harder questions, to help you see if you're *really* ready, are at <http://www.math.unb.ca/ready/exercise.html>.

## 1.7 HELP! They want to take my calculator away!

In some first-year calculus courses, calculators are not permitted. To be exact, they do not permit them in the pretest, tests, or exams; you can certainly have one in the lecture if you want, or use one for your homework. In other introductory calculus courses, calculators or computers are used intensively to explore the behaviour of functions. You should find out the policy at the university you plan to attend!

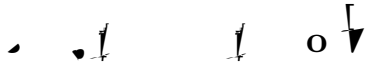
- If your calculus course has a no-calculator policy, the questions will be de-



GO O

## **Getting a copy free**

Please note, this is the truly cool way to get this booklet (and saves us time .



In this section, we present over 100 questions that cover the absolute minimum of mathematical skills that you will need for university calculus. We give worked answers for a few; the answers to the rest are in the back of the booklet.

There are important topics in high school mathematics that are not covered here, and there may even be some things you'll need for calculus that we have missed. A good student should be able to answer significantly more challenging problems than these. However, if you understand all of this material, you will be reasonably well prepared for calculus.

### 3.1 Arithmetic

You should be able to do basic arithmetic without a calculator, including operations on fractions, negative numbers, and decimals. You should be able to compute simple powers and roots. This material, which is from the elementary and junior high school curriculum, is fundamental for everything that follows. Also, many arithmetic problems are really “algebra with numbers”.

#### Examples:

1. Find  $\frac{1}{3} + \frac{2}{3}$ .

**Solution:** To add (or subtract) fractions, you must first find a common denominator:

$$\frac{1}{3} + \frac{2}{3} = \frac{1 \cdot 1}{3 \cdot 1} + \frac{2 \cdot 1}{3 \cdot 1} = \frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$$

2. Convert 1

**Problems:** (answers on page 0

1. Find  $\frac{2}{3} + \frac{3}{5}$
2. Find  $\frac{3}{4} - \frac{1}{2}$
3. Find  $\frac{1}{3} \times -$  in lowest terms.
  - . Convert  $3 \frac{12}{100}$  to a fraction in lowest terms.
  - . Convert  $3 \frac{2}{10}$  to a decimal.
  - . Convert  $1 \frac{10}{100}$  to a decimal.
  - . Find  $0.0001 \times 0.010001$
  - . Find  $1.23 \times 0.1$
9. Find  $1.00 + 9.99$
10. Find  $1 - (-2)$
11. Find  $\sqrt{\quad}$
12. Find  $20^3 \cdot 20^5$  as a decimal.

## 3.2 Basic Algebra

As observed above, basic algebra is closely related to arithmetic, and many of its rules are familiar as “rules of arithmetic”. You should be aware that  $a \times b$  may be written as  $ab$ .

You should know the basic rules for addition, subtraction, multiplication, division, and exponents, and be aware of the operations such as division by 0 and taking the square root of a negative number that cannot be done within the real number system.

You should know how to solve a simple equation, simplify an algebraic expression, and evaluate an expression by plugging values into it. All of these will be very important when you take calculus.

### Examples:

1. Simplify  $\frac{7^3}{4^4}$ .

**Solution:** By basic rules of exponents

$$\frac{7^3}{4^4} = 7^{-4} 3^{-4} = 3^{-1} = \frac{3}{1}$$

2. Solve  $x + 2 = 5$  for x when  $x = 3$ .

**Solution:** Substituting  $x = 3$  and adding, subtracting and dividing like quantities on both sides of the equation we reduce to an equation which is its own solution:

$$\begin{aligned} & x + 2 = 5 \\ & x - 2 + 2 = 5 - 2 \\ & 3 + -2 = 3 - 2 \\ & 3 \left(\frac{1}{3}\right) = -9 \left(\frac{1}{3}\right) \\ & -3 \end{aligned}$$

so  $x = -3$

**Problems:** (answers on page 0)

1. Simplify  $(4 - 2)^2$
2. Simplify  $x^2 - 4x + 4$ , factoring if possible.
3. Simplify  $\frac{\frac{y}{x} + \frac{2}{x}}$ .
  - . Simplify  $(x^2 + 3x + 4)$ , factoring if possible.
  - . Solve  $3x + 2 = 2x + 2$  for x.
  - . If  $x = 2$  and  $y = 3$ , find  $x^2 - y + 3$ .
  - . Simplify  $\frac{x^2 - (-2)}{2}$
  - . If  $x = 2$  and  $y = 2$ , find  $x^2 - y + 2$
9. Solve  $x^5 + 2 = 3x^5 - 1$ .
10. If  $x = 3$  and  $y = 1$ , find  $x^2 - y$ .
11. Simplify  $(x^2 - 2)(-2x^2)$ .
12. Solve  $x + 2 = x - 4$ .

### 3.3 Inequalities and absolute values

You should be able to solve simple inequalities and perform algebraic operations with them. In particular, you should know which operations reverse inequalities and which ones preserve them. You should understand interval notation, including open, closed, and half-open intervals, and intervals with limits at  $\infty$ . You should know how to compute an absolute value, and to do simple algebra using the absolute value function.

#### **Examples:**

1. Solve

**Problems:** (answers on page 0

1.  $f$  ,  $\geq$  , and





**Problems:** (answers on page 0)

1.  $f(x) = \sqrt{x-1}$ , find the largest possible domain (within the real numbers) on which  $f$  could be defined.
2.  $f(x) = \sqrt{x+1}$ , find the domain of  $f$ .
3.  $f(x) = x^2 + 2$ , find  $f(1)$ .
4.  $f(x) = x^2$ , find the range of  $f$ .
5. Find the domain and range of  $f(x) = \frac{1}{x}$ .
6.  $f(x) = \frac{1}{x+1}$ , find (a)  $f(2)$ ; (b)  $f(-2)$ ; (c)  $f(\sqrt{x})$ .
7.  $f(x) = \sqrt{x-1}$  and  $g(x) = x^2$ , find  $(f \circ g)(x)$ .
8.  $f(x) = \sqrt{x-1}$  and  $g(x) = x^2$ , find  $(g \circ f)(x)$ .
9.  $f(x) = \sqrt{x+1}$ , find  $f^{-1}(x)$ .
10.  $f(x) = x^{-3}$ , find the inverse function.
11. Which of the following functions is its own inverse:
  - (a)  $f(x) = x^2$ , (b)  $f(x) = -x$ , (c)  $f(x) = \frac{1}{x}$
  - (d) each of (a), (b) and (c) (e) (a) and (c) only
12. Which of the following functions has an inverse function
  - (a)  $f(x) = x^3$ , (b)  $f(x) = x^5$ , (c)  $f(x) = \sqrt{x}$
  - (d) each of (a), (b) and (c) (e) (a) and (c) only
13. Find the inverse of  $f(x) = \frac{x+1}{2x+1}$ .

### **3.5 Polynomials**

You should know how to add, subtract, multiply, divide, and factor polynomials. You should know special forms such as the difference of powers. You should understand the connection between roots and factorizations, and be able to solve a

**Problems:** (answers on page 0)

1. Solve:  $x^2 - 5x + 3 = 0$ .
2. Factorize  $2x^2 + 3x + 2$ .
3. Put  $x^2 + 2x + 2$  in completed square form, and graph it.
  - . Simplify:  $(x^2 + 3)(x^2 + 1) - 4$ .
  - . If we divide  $x^4 + 3x^3 + 2x^2 + x + 1$  by  $x + 1$ , what is the remainder
  - . Expand:  $(x + 2)^4$ .
  - . Expand and simplify:  $\frac{(x + 1)^3 - (x - 1)^3}{x}$
  - . Expand and simplify:  $\frac{(x + 1)^3 + (x - 1)^3}{x}$
9. Expand:  $(x^2 - 1)(x^2 - x + 1)$
10. How many real solutions does  $x^4 - x^2 + 1 = 0$  have
11. Factorize:  $x^4 - 13x^2 + 3 = 0$

### 3.6 Algebra with Fractions

You should be able to simplify a fractional expression, convert a “stacked” fractional expression into a simple one, put fractional expressions over a common denominator, and perform a partial fraction expansion. These skills will be useful in finding various derivatives, simplifying derivatives and integrals, and in particular for the “partial fractions” technique of integration.

#### Examples:

1. Simplify  $\frac{(x-1)^2 + 1}{x+1}$ .

**Solution:**  $\frac{(x-1)^2 + 1}{x+1} = \frac{x^2 - 2x + 1 + 1}{x+1} = \frac{x^2 - 2x + 2}{x+1} = \frac{x^2 + 2x + 1 - 4x + 1}{x+1} = \frac{(x+1)^2 - 4x + 1}{x+1} = (x+1) - \frac{4x-1}{x+1} + 1$

2. Simplify  $\frac{\frac{x-1}{x-2} - \frac{x-2}{x-1}}$ .

**Solution:**  $\frac{\frac{x-1}{x-2} - \frac{x-2}{x-1}}{\frac{3}{(x+1)(x+2)}} = \frac{\frac{(x-1)(x-1) - (x-2)(x-2)}{(x-2)(x-1)}}{\frac{3}{(x+1)(x+2)}} = \frac{(x^2 - 2x + 1) - (x^2 - 4x + 4)}{(x-2)(x-1)} \cdot \frac{(x+1)(x+2)}{3} = \frac{2x - 3}{(x-2)(x-1)} \cdot \frac{(x+1)(x+2)}{3}$

3. Expand  $\frac{x^2 + 1}{x^2 - 1}$  in partial fractions.

**Solution:**  $\frac{x^2 + 1}{x^2 - 1} = 1 + \frac{2}{x^2 - 1} = 1 + \frac{2}{(x+1)(x-1)}$ .

We want to write this as  $1 + \frac{A}{x-1} + \frac{B}{x+1}$  for suitable  $A$  and  $B$ . Putting the fractions over a common denominator, we have  $A(x+1) + B(x-1) = 2$ ; so  $A = 1$ ,  $B = 1$ , and

$$\frac{x^2 + 1}{x^2 - 1} = 1 + \frac{1}{x-1} + \frac{1}{x+1}$$



### 3.7 Rationalizing numerators or denominators

You should know how to eliminate squares (and other roots) from the numerator or denominator of a fraction by multiplying both the numerator and denominator by an appropriate expression. This technique will be important in finding the derivatives of certain expressions involving roots.

#### Examples:

1. Rationalize the denominator of  $\frac{1}{\sqrt{a}}$

**Solution:** By multiplying both numerator and denominator by  $\sqrt{a}$  we obtain

$$\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{a}} \frac{\sqrt{a}}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

2. Rationalize the denominator of  $\frac{1 + \sqrt{a}}{2 + \sqrt{a}}$

**Solution:** Recall, when we multiply  $(a + b)(a - b)$  we obtain a difference of squares  $a^2 - b^2$ . So, if the denominator of a rational expression contains a constant added to a square root term, we can eliminate the root term by multiplying by the difference of the constant and the root term. In this case if we were to multiply both the numerator and the denominator by  $2 - \sqrt{a}$  we will obtain the desired result.

1 +

**Problems:** (answers on page 0

1. Rationalize the denominator of  $\frac{+1}{\sqrt{\quad}}$
2. Rationalize the denominator of  $\frac{\quad}{\sqrt{\quad}\sqrt[3]{\quad}}$
3. Rationalize the denominator of  $\frac{1}{\sqrt{\quad} + \sqrt{\quad}}$
4. Rationalize the denominator of  $\frac{-\sqrt{\quad}}{-\sqrt{\quad}}$
5. Rationalize the denominator of  $\frac{1}{1 + \sqrt[1]{\quad} + \sqrt[2]{\quad}}$
6. Rationalize the numerator of  $\frac{\sqrt{\quad} + \sqrt{\quad}}{\quad}$
7. Rationalize the numerator of  $\frac{-\sqrt{\quad}}{-\sqrt{\quad}}$
8. Rationalize the numerator of  $\frac{\sqrt{\quad+1} - \sqrt{\quad}}{\quad}$
9. Rationalize the numerator of  $\frac{\sqrt[3]{\quad+} - \sqrt[3]{\quad}}{\quad}$

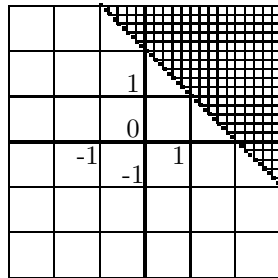
### **3.8 Linear Graphs**

You should be able to graph linear functions and inequalities, determine the slope



**Problems:** (answers on page 0)

1. Find the slope of the line  $2x - 2$
2. Give the equation of a line with slope 3, through  $(0, 0)$ .
3. If a line has slope  $3/2$  and passes through  $(2, 2)$ , give its equation.
  - . Find the distance from the point  $(0, 3)$  to the point  $(3, 0)$
  - . Find the slope of the line through the points  $(0, 3)$  and  $(3, 0)$ .
  - . Find the point of intersection of the lines  $x - y = 1$  and  $2x + 2y = 2$
  - . Find the equation of the line with slope  $-1/2$  and  $y$ -intercept 3
  - . Find the slope of a line orthogonal to the line  $3x + y = 1$ .
9. Find the equation of the line through  $(3, 2)$  and  $(1, 0)$
10. Which inequality is this the graph of



- (a)  $2x - 2$  (b)  $2x + 2$  (c)  $2x + 2$  (d)  $2x - 2$  (e)  $2x + 2$
11. Which of these lines is parallel to  $2x - 1$ 
    - (a)  $2x - 1$  (b)  $2x + 1$  (c)  $3x + 1$
    - (d)  $2x - 3 + 2$  (e)  $2x - 1 + 1$
  12. Which of these lines is orthogonal to  $3x - 1$ 
    - (a)  $3x - 1$  (b)  $3x + 1$  (c)  $3x + 1$
    - (d)  $3x + 2$  (e)  $1 - 3x$



**Solution:** (f . raph B does not have a uni ue value for every , and the



**Problems:** (answers on page 0)

1. Simplify  $\frac{\sqrt{x} \times 2}{1 - 2}$

2. If  $x^{3/2} = 2\sqrt{x} - b$ , what is

3. Solve:  $x2^x = 2$ .

4. Solve:  $2^0 + 2^x = 9$ .

5. Find  $\left(\frac{1}{\sqrt{1}}\right)^{1/2}$

6. If  $a^2 = b^2$ , for real numbers  $a, b$ , which of the following must hold

### 3.11 Logarithms

Logarithms are extremely important in many of the sciences, and it is important to be able to differentiate and integrate expressions using them. To do that, you have to be able to manipulate logarithms algebraically.

You should know the definition of logarithms to various bases, their relation to powers and roots, and the change-of-base formula  $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$ . While logarithms appear tricky at first, there are actually not very many things to learn about them.

#### **Examples:**

1. Find  $\log_5(12)$ .

**Problems:** (answers on page 0)

1. Find  $\log_{10}(0.001)$ .
2. Find  $\log_2(\quad) + \log_3(9)$
3. If  $3^x = \log_x(3)$ , which is true  
(a)  $x = 3$  (b)  $x = 1/3$

### 3.12 Geometry and basic trigonometry

Calculus does not use very advanced geometry, but you should be thoroughly familiar with similar triangles, Pythagoras' theorem, and parallel lines; and, from analytic geometry, the midpoint and distance formulae, and the "negative reciprocal" rule. Trigonometry is important in various branches of science, but especially in mathematics, physics, and engineering.

You should be able to convert between degrees and radians ( $180^\circ = \pi$  radians). You should know the definitions of the trig functions, and be able to use them to find sides and angles of triangles. You should know and be able to use the sine and cosine laws for triangles.

Most angles do not have trig functions that are easy to give as exact expressions rather than decimal approximations (and you will not be expected to do so), but you should know the trig functions of a few common angles, such as  $0^\circ$ ,  $30^\circ$ ,  $45^\circ$ ,  $60^\circ$ , and  $90^\circ$ . You should also know how to find the trig functions of angles outside the range  $0^\circ$ – $90^\circ$  in terms of trig functions of angles in that range.



**Problems:**



**Problems:** (answers on page 0)

1.  $\sin(2\theta)$

- (a)  $2 \sin(\theta) \cos(\theta)$  (b)  $\sin(\theta) + \cos(\theta)$  (c)  $\cos(\theta) - \sin(\theta)$   
 (d)  $2 \cos(\theta) - 2 \sin(\theta)$  (e)  $2 \cos(\theta) \sin(\theta)$

2.  $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

- (a)  $\cos(\theta) \sin(\theta)$  (b)  $\sin(\theta)$

(c)  $\frac{1}{\sin(\theta)}$  (d)  $\cos(\theta)$  (e)  $1$

3. If  $\cos(\theta) = \sqrt{2}/2$ , then  $\tan(\theta)$

- (a)  $2$  (b)  $1$  (c)  $0$  (d)  $1/2$  (e)  $\pm 1$

4. If  $\tan(\theta) = 3$ , then  $\cot(\theta)$



**Problems:** (answers on page 10)

- If the area of a triangle is  $100 \text{ cm}^2$  and its base is equal to twice its height, what is its base (a) 5 cm (b) 10 cm (c) 20 cm (d) 40 cm (e) 200 cm
- If the perimeter of a rectangle is 24 meters, and the diagonal is 10 meters, what is the area
- If the perimeter of a rectangle is 24 meters, and the area is 30 square meters, what is the diagonal
  - A right triangle has area 30 square meters and perimeter 30 meters. What is its hypotenuse
  - If a goat can eat the grass on a lawn in 1 day and a sheep can eat the grass on the same lawn in 2 days, how long would it take two goats and a sheep working together. Assume the grass is not growing.  
(a) 2/3 of a day (b) 3/4 of a day (c) 1 day (d) 2 days (e) 3 days
  - Joe started day trading with a certain amount of money. On the first day he doubled his money, and on the second day he lost  $2/3$  of what he had at the end of the first day. At the end of the second day he had \$100. How much did he start with  
(a) \$50 (b) \$75 (c) \$100 (d) \$120 (e) \$133.33 (f) \$150
  - If a puppy weighs a third as much as a dog, and together they weigh 12 kg., how much does the puppy weigh
  - A driver is driving 100 km. She drives the first 40 km at 40 km/hr and the second 60 km at 60 km/hr. How long does the trip take (a) 30 minutes (b) 40 minutes (c) 50 minutes (d) 60 minutes (e) 90 minutes
- The towns of Ayton and Beaton are 60 km apart. At noon, Alice leaves Ayton and drives to Beaton at 40 km/hr, and Bob leaves Beaton and drives (on his tractor) to Ayton at 30 km/hr. At what time do they meet
- The towns of Ayton and Beaton are 60 km apart. At noon, Alice leaves Ayton and drives to Beaton at 90 km/hr. If Bob rides from Ayton to Beaton on his bicycle at 15 km/hr, when must he leave Ayton to get to Beaton at the same time that Alice does

### 3.14159265358979... **Harder Problems**

The material earlier in this booklet represents the minimum that you need to know before you begin university calculus. We hope that it won't be *all* that you know! You should also know something about linear algebra, geometry, statistics, and other areas of mathematics; you should have experience applying mathematics in other subjects; and you should be able to write clear explanations of what you know, and solve problems that require a certain amount of lateral thinking. In this section, we present some problems intended to challenge the stronger student. Such students may also want to look for problems in publications such as *Crux Mathematicorum*, or compete in mathematics contests.

Answers to these questions are *not* given.

1. Find all solutions to

9. Which is larger,  $100^{200}$  or  $200^{100}$ ? Explain your answer.
10. For  $n = 1, 2, 3, \dots$  we define  $n!$  as  $n \times (n-1) \times \dots \times 3 \times 2 \times 1$ . (This is pronounced “ $n$  factorial” and is important in combinatorics, probability, and other areas of math.)
- (a) Find  $n!$  for  $n = 3$
- (b) How many 0’s does  $100!$  end in?
11. By using Pythagoras’ theorem and elementary geometry, find  $\sin(30^\circ)$ ,  $\sin(45^\circ)$ , and  $\sin(60^\circ)$ .
12. Determine  $\sin(15^\circ)$  and  $\sin(75^\circ)$ .
13. Starting with the sum-of-angles formulae for sines and for cosines, derive the difference-of-angles formulae, the double-angle formulae, the half-angle formulae, and the formulae for  $\sin(\alpha \pm \beta)$ ,  $\cos(\alpha \pm \beta)$ .
14. A rhombus has the edge length equal to its short diagonal. Find the area in terms of the length of the *long* diagonal.
15. A river runs straight east and west. A horse and rider are 5 km north of the river. The rider could get to camp by riding 2 km south and then 2 km west, but the horse needs to get to the river to drink, too. What is the shortest distance that the horse and rider must travel to get first to the river and then to the camp?
16. The decimal expansion of  $\frac{1}{3}$ ,  $0.333\dots$ , repeats with period 1; and that of  $\frac{1}{7}$ ,  $0.142857142857\dots$  repeats with period 6. Find fractions whose decimal expansions repeat with periods 2, 3 and 4. Can you find the *smallest* denominators that work?
17. Show (without calculus) that for any positive  $x$ , we always have

$$\frac{x+1}{2} \geq \sqrt{x}$$

## Answers to problems:

**Section 3.1** (1)  $\frac{1}{15}$  (2)  $\frac{11}{20}$  (3)  $\frac{2}{5}$  (4)  $\frac{25}{5}$  (5) 1 (6) 1 (7) 0 001  
(8) 0 123 (9) 11 0 (10) 3 (11) (12) 0 002

**Section 3.2** (1) 4



**Section 3.12** (1  $90^\circ$  (2 e (3 c ( d ( a (  $-1\sqrt{2}$  or  $-0.0$   
(  $^\circ$  (  $\sqrt{3}$  (9  $\pi/10$  or  $\pi/12$  (10 c (11 c (12  $1/\sqrt{3}$  (13  
b (1 d

**Section 3.13** (1 e (2 c (3 e ( e ( b ( c ( b (  $-2$   
(9 c (10 a

**Section 3.14** (1 c (2  $2\text{ m}^2$  (3  $\sqrt{109}\text{ m}$  (  $13\text{ m}$  ( a ( f (  $3\text{ kg}$   
( d (9 12:30 PM (10 : 0 AM

**Some useful facts:**

Algebraic identities	
$a^2 - b^2 = (a + b)(a - b)$	$(a + b)^2 = a^2 + 2ab + b^2$
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$(a - b)^2 = a^2 - 2ab + b^2$
Exponential identities	
$b^x \cdot b^y = b^{x+y}$	$\frac{b^x}{b^y} = b^{x-y}$
$(b^x)^y = b^{xy}$	$b^{-x} = \frac{1}{b^x}$
$\sqrt[a]{b} = b^{1/a}$	$(\sqrt[a]{b})^x = b^{x/a}$
$\ln(b^x) = x \ln(b)$	$\log_e(b^x) = x \log_e(b)$



## Getting a copy free

Please note, this is the truly cool way to get this booklet (and saves us time .

- This document may be downloaded in Adobe PDF or PostScript format from the web, at our website <http://cs.smu.ca/apics/calculus/welcome.php> . This site also contains links to free software that will allow you to display and print the document.